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The intensification of hurricanes

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An account is given of the dynamic balance and the thermodynamic considerations which underlie the intensification of a tropical disturbance into a mature hurricane. In particular, it is deduced that the *e*-folding time for the later stages of the intensification process is of the order of 16 h but that the eye formation itself can occur on a much shorter time scale.

1. Introduction

Once in every several days in the summer months of the northern hemisphere a very weak disturbance drifts off the western coast of Africa at about 15° latitude and proceeds westward, intensifying into a tropical depression (a configuration in which the maximum particle speed is about 25 miles per hour and whose diameter is of the order of 1000 miles). A few of these tropical depressions grow into hurricanes and it now seems certain that all hurricanes do evolve from the foregoing beginnings. In this paper, we suggest that the dynamics and thermodynamics which underlie the intensification of such disturbances into hurricanes are intrinsically the same as those which were invoked earlier to explain the maintenance of a mature hurricane (Carrier, Hammond & George 1971).† In this section we give a qualitative description of the processes which make the storm grow; in subsequent sections we provide quantitative estimates of the time scale with which the storm develops.

If, over the ocean in the neighbourhood of 15° latitude, the large mass of air depicted in regions I and II of figure 1 has the circumferential velocity distribution indicated in that figure, then the boundary layer in region III of that figure must transport fluid radially inward and this fluid must be supplied by a very slow downward motion of the fluid in region I. Conversely, the swirl distribution in region II is such that fluid must be transported upward from the boundary layer labelled IV into region II. Suppose that figure 1 describes the configuration at time t_1. Then, at t_2, with $t_2 > t_1$, the configuration will have changed to that of figure 2. In figure 2, the interface, C–C, between ambient air which has been pushed up by incoming air from IV and the new air itself is higher than it was in figure 1. The amount of that change in height depends both on the amount of new air supplied during $t_1 < t < t_2$ and on the amount by which $R(t)$ has changed (decreased) during that time interval. By the time that this storm has intensified so much that the upward motion is the most important

† Henceforth referred to as (CHG).

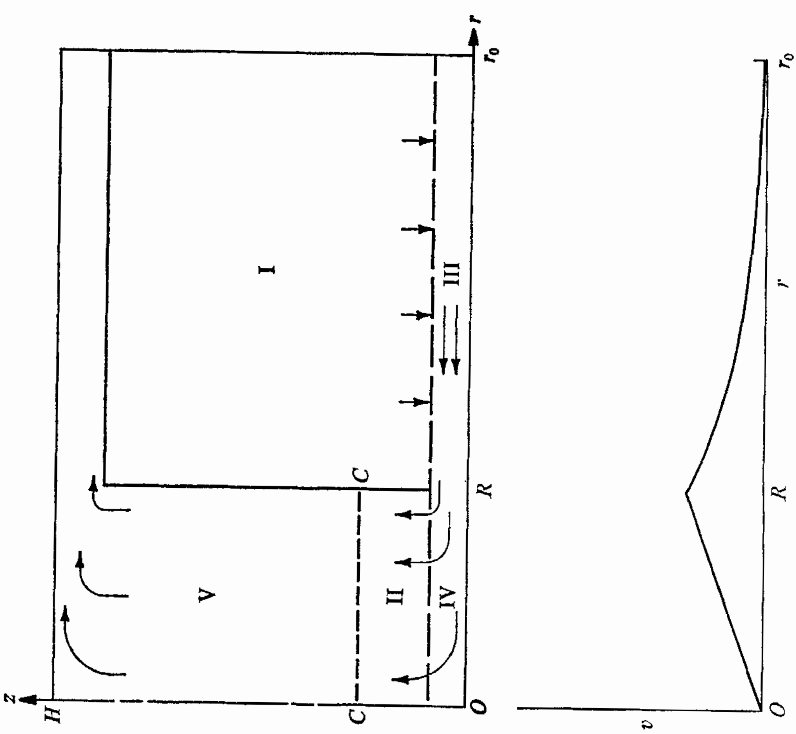

FIGURE 1

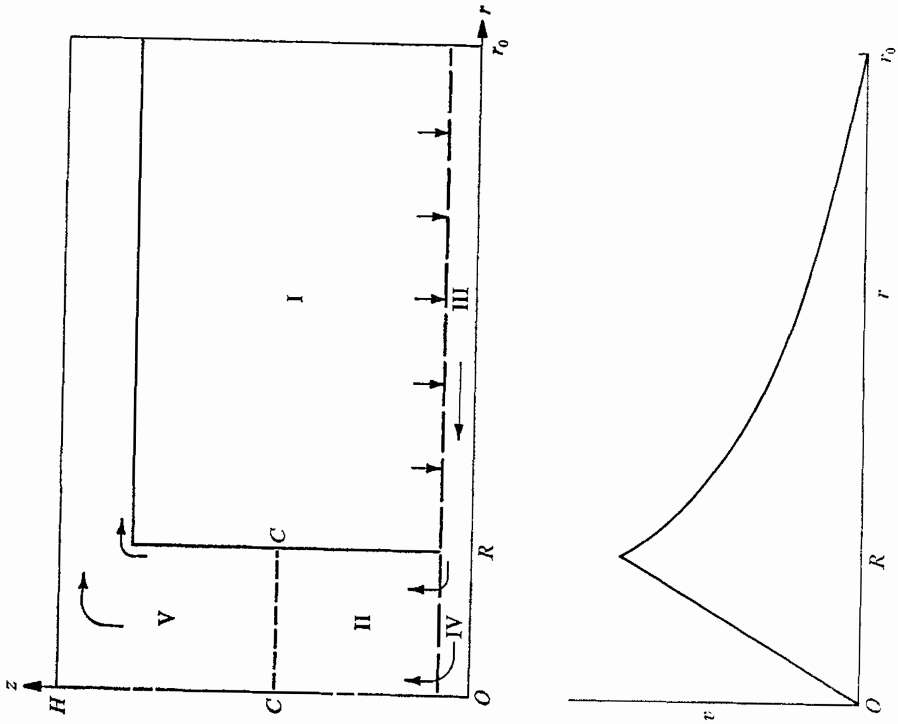

FIGURE 2

mechanism in the vertical transport of heat, this air below C–C will have a thermodynamic state at each level which is achieved as follows. Low altitude, warm, moist air from the boundary layer III expands adiabatically; the moisture condenses so that the moisture content at any point in its history is the saturation content associated with the temperature at that point. The condensate falls as rain but the heat of condensation is retained by the air. Henceforth, we refer to such a sequence of states as a moist adiabat, the name conventionally used by meteorologists. Before the updraft becomes so strong that this description prevails, the state of the air at a point on the centreline below C–C will be intermediate between the moist adiabatic state (for that pressure) and the ambient state.

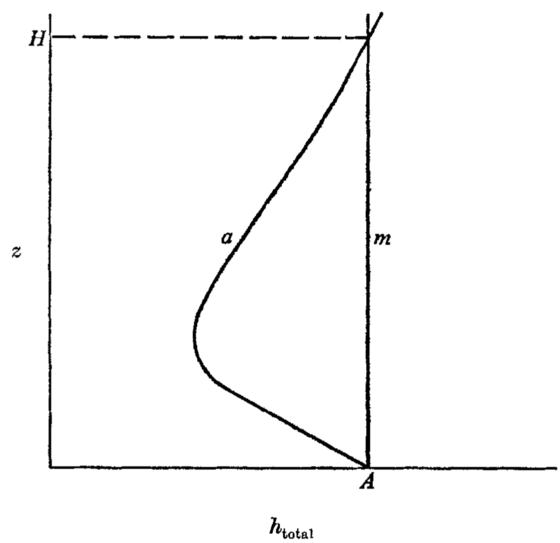

FIGURE 3

Under the conditions which prevail over the tropical oceans this air has, at each altitude z, a higher total enthalpy and a smaller density than the air it has replaced. By total enthalpy, h_0, we mean

$$h_0 = C_p T + gz + \tfrac{1}{2}q^2 + Lm,$$

where q is particle velocity, L is latent heat of vaporization and m is mass of H_2O vapour per unit mass of air.† Furthermore, much of the air above C–C which, at time t_2, is higher in altitude than it was at t_1, has a smaller density at t_2 than the air at the same place had at t_1. These statements are merely consequences of the fact that the total enthalpy distribution in the ambient atmosphere resembles that of curve a in figure 3, whereas the distribution which would exist if saturated sea-level air were expanded along a moist adiabat to higher altitudes is shown by curve m.

In a way which is discussed quantitatively later, the foregoing comparisons imply that a column of air (of unit cross-section) at $r = r_0$ weighs more than the

† Except for the very small contribution $\frac{1}{2}q^2$, h_0/C_p is the 'equivalent potential temperature' of the meteorologist.

column of air at $r = 0$ in figure 1 which, in turn, weighs more than the column of air at $r = 0$ in figure 2. Thus there is a pressure discrepancy $p(r_0, 0, t) - p(0, 0, t)$ which, in the inviscid region above the boundary layers, must be balanced by the radial acceleration of the fluid. Furthermore, since, for all practical purposes, $p(r_0, 0, t)$ does not change with time, the amount of radial acceleration at time t_2 must be greater than it was at time t_1. Anticipating that the major contribution to the radial acceleration is the centripetal acceleration, this implies that the circumferential velocities at time t_2 must have a larger integral over r than at time t_1. But the particles of swirling fluid above the boundary layer conserve their angular momentum and the circumferential velocity can have increased only if the particles have moved radially inwards. Thus, $R(t_2)$ is indeed smaller than $R(t_1)$.

If figures 1 and 2 were more than schematic, the particle paths shown near the bottom of the inner region would have to be consistent with density, pressure and velocity distributions which are in equilibrium proportions. Whatever those details, however, the particles rising near the centre start their ascent from virtually sea-level conditions ($T = T_{\text{sea level}}$ and saturated), whereas particles further out begin their ascent from a level which has less enthalpy and they do not drop as much in density with decreasing pressure. This, of course, is consistent with the relative velocities depicted in figures 1 and 2, but it certainly does not imply that the interface C–C is, and remains, horizontal. As we shall see, this is not of any real importance in the general understanding of the phenomenon but such detailed balance requirements would underlie a more meticulous analysis of the flow details.

If the foregoing process were quasi-steady, i.e. if each successive configuration were almost a steady equilibrium configuration, one could infer from a steady boundary-layer theory the intensity of the down flow into region III for the velocity field at time t in I, infer from that the rate at which mass is being supplied to region II at that time, infer the rate of change of central pressure implied by this 'flushing rate', and require that the pressure-drop history $p(r_0, 0, t) - p(0, 0, t)$ so implied be consistent with the radial acceleration at each time t. Such a sequence of steps would lead to a description of the evolution in time of the flow; clearly it would lead, eventually (say at time t_3) to a flow field in which $p(0, 0, t_3)$ is given by the weight per unit area of the column of fluid on the axis of the system whose thermodynamic states lie on the appropriate moist adiabat.

However, the process is *not* really quasi-steady. The time scale on which this sequence of events would evolve is much shorter than the time required to equilibrate changes in the boundary-layer flow and an unsteady analysis of that boundary layer must be included. With that modification (since no other transient effects are important until after the central region has been filled with moist adiabat air) the foregoing analysis is appropriate and we shall carry it out in the remaining sections of this paper. We must do more than that, however, since the foregoing evolution does not provide for the formation of the quiescent, warm, dry eye of the storm. The details of that process are more complicated than the details of the foregoing mechanisms but the idea is still simple.

When C–C, the interface between ambient air and updraft air, reaches H,

dR/dt is not zero. Furthermore, the inertia of the system is far from negligible and R will continue to get smaller even though the central pressure cannot continue to decrease. The overshoot in the pressure drop, $p(r_0, 0, t) - p(0, 0, t)$, needed to support the centripetal acceleration will be taken up by a decrease of the inward radial velocity component of the particles; this acceleration in the positive r direction will eventually lead to a reversal of dR/dt and R will get larger again. As it does so, air must refill the enlarging region II and the only air available for this task is that near H. That is, air in the state $p(H, t)$ will be recompressed adiabatically as it flows down into region E, as is depicted in figure 4.

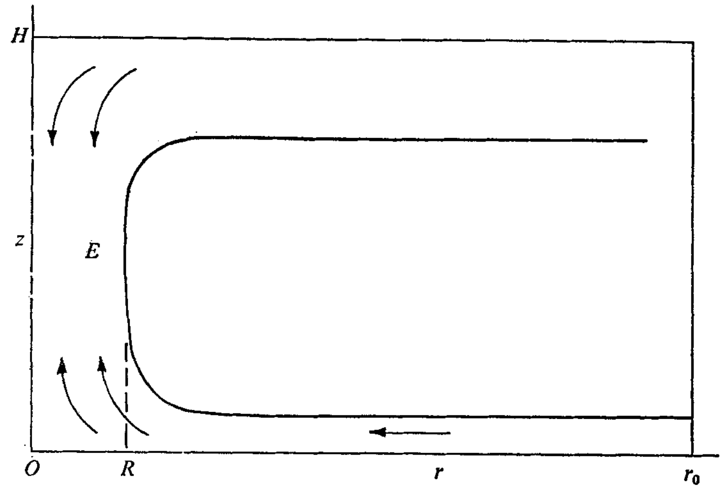

FIGURE 4

It does not really matter whether this air is primarily air which has recently risen in the central part of the inner region or whether it is high altitude air which has been there longer (as was once suggested by Malkus (1958)), but it *must* be dry air with very little angular momentum because no other air is available nearby.

When figure 4 prevails, the central column is lighter (in weight) than the column of moist adiabat air which previously occupied that column and the sea-level central pressure is lower than it was. With such a low central pressure, the configuration can settle down in a state which, schematically again, is not very different from that of figure 5. The entrainment of air in E by the rising air and the slow circulation in E which is so implied can keep E supplied with warm dry air and negate the transport processes which tend to cool that air. The pressure gradient across E will be negligibly small so that no swirl is supported and no updraft near the bottom is implied; thus, generally, the steady-state conditions described in (CHG) will prevail. In particular, the central sea-level pressure will lie somewhere between that which is consistent with a core of dry recompressed air (which has gone from sea-level conditions moist adiabatically to H and then, dry, to its position in E) and that whose states lie on the sea-level condition moist adiabat. Thus, the steady picture presented in (CHG) emerges as a direct consequence of a process in which air moves radially inward to provide dynamic balance with a changing pressure drop implied by the filling of the central column with spinning moist adiabat air. This evolution overshoots because of the inertia of the system and, in the rebound, dry air from the top of the domain is drawn down the centre where, because it had small angular momentum, it provides a stable configuration in which the eye is relatively warm, dry and motionless.

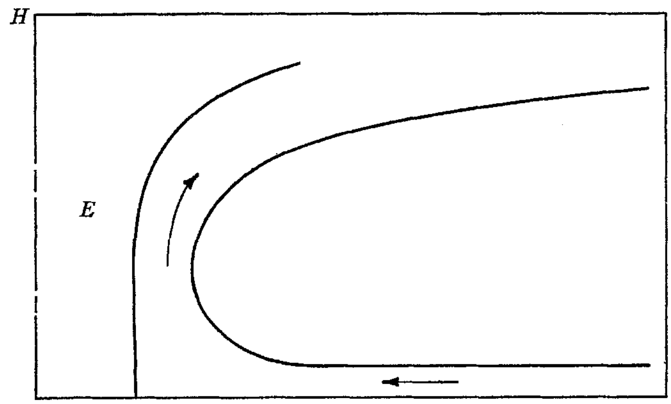

FIGURE 5

One other point deserves discussion before we turn to quantitative details. When the storm is still very weak, the updraft in the central region is *very* slow. Accordingly, the cumulus convection, radiation, and turbulent mixing which determine the ambient state of the atmosphere can compete rather effectively with the organized vertical convection of heat which is implied by the upflow. This implies that, in this very early stage, the irregularities in the state of the ambient atmosphere, the sea surface, and the turbulent motion could readily negate the effects of organized vertical convection and the storm could fail to develop further. When, on the other hand, the processes described in the foregoing have developed to that stage where the organized vertical transport is so rapid that it dominates the ambient processes, the storm must continue to intensify into a full-fledged tropical cyclone. The threshold beyond which the storm always intensifies would be hard to specify. Little is known quantitatively about the variations in those items which control the ambient state of the atmosphere and one cannot easily estimate the intensity of the updraft which would be needed to render them locally ineffectual.

We turn now to some details.

2. The dynamics

The dynamics of the air whose motion is almost horizontal throughout most of the hurricane region are implied to a sufficient degree of accuracy by

$$u_t + uu_r + wu_z - 2\Omega v - \frac{v^2}{r} + \frac{1}{\rho} p_r = \nu u_{zz}, \tag{2.1}$$

$$(rv)_t + u(rv)_r + w(rv)_z + 2\Omega ru = \nu(rv)_{zz}, \tag{2.2}$$

$$p_z + \rho g = 0 \tag{2.3}$$

and

$$rw_z + (ru)_r = 0. \tag{2.4}$$

The cylindrical co-ordinate system is fixed in the rotating earth and Ω is the normal component of the earth's angular velocity at the latitude of interest. At 15° latitude, $\Omega \simeq \frac{1}{16}$ rad/h. Our objective is to construct a flow field in $0 < r < r_0$ whose general features are those described in the previous section.

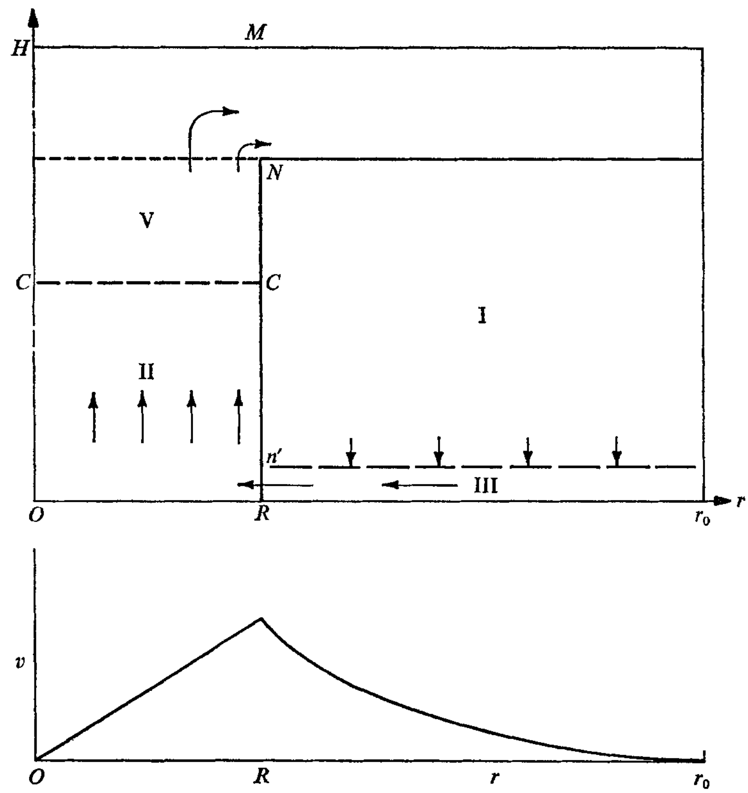

FIGURE 6

More meticulously, we require that $u = v = 0$ at $r = r_0$ and $u = v = w = 0$ at $z = 0$; we also require that u and v be small at early times. In this construction, we infer in detail only those items whose accurate evaluation is crucial to the understanding, in the large, of the phenomenon. The analysis of other items will be very informal indeed.

We envisage a flow in region I of figure 6 in which the vertical velocity w is slow and more or less independent of r, in which the radial velocity u is negative and in which the circumferential velocity increases with time. The latter evolution occurs because the angular momentum of the air mass as measured in an inertial system is not zero and this momentum is conserved as particles move radially toward the line $r = 0$ which is the centre of our storm.

Accordingly, we adopt the flow field† in region I in which

$$\Psi = rV(r,t) = \Omega[R_0^2 - R^2(t)]\,\frac{r_0^2 - r^2}{r_0^2 - R^2}, \tag{2.5}$$

$$\Phi = rU(r,t) = R\dot{R}\,\frac{r_0^2 - r^2}{r_0^2 - R^2}, \tag{2.6}$$

$$W_1(r,z,t) = \frac{2R\dot{R}}{r_0^2 - R^2(t)}\,z, \tag{2.7}$$

where we use capital letters in order to save u, v, w for the flow in region II.

It is easily verified that this flow field is consistent with (2.2) and (2.4) and that it implies, uniquely, a radial pressure gradient through (2.1). Discussion of the internal consistency of that pressure field with (2.3) comes later.

It is also easily verified that, when $R(t_0) = R_0$, there is no swirl in $R_0 < r < r_0$, that particles which were at R_0 when $t = t_0$ are subsequently at $R(t)$, and that the swirl intensifies as R gets smaller. In particular, it is realistic to study the overall flow field which emerges when

$$r_0 = O(1000\text{ miles}), \quad R_0 = O(200\text{ miles}), \quad \tfrac{1}{12}R_0 \leqslant R(t) \leqslant R_0.$$

Thus far, the relevance of (2.5), (2.6) and (2.7) to the hurricane problem is conjectural and we must demonstrate that relevance when we have deduced the estimates which are needed for that demonstration.

The time-dependent boundary layer under the inviscid flow described by (2.5), (2.6) and (2.7) has already been studied (Carrier 1971). For the numbers adopted above and with

$$R_0^2 - R^2(t) = R_0^2 e^{\alpha t},$$

the flow into the boundary layer and, in particular, the transport of fluid by that boundary layer into the region $r < R(t)$ has been determined. The volume transport, $Q(t)$, found in that study is given by

$$Q(t) \simeq \left(1 - \frac{6\alpha}{7\Omega}\right) Q_0 e^{\alpha t}, \tag{2.8}$$

where Q_0 is given by

$$Q_0 = \pi(r_0^2 - R^2)\,(R_0^2/r_0^2)\,(\nu\Omega)^{\frac{1}{2}}.$$

If the transient process were of no importance, Q would be given by $Q_0 e^{\alpha t}$; thus, the factor $(1 - 6\alpha/7\Omega)$ describes the extent to which the filling of the central column with moist adiabat air is inhibited by the transient character of the boundary layer.

† Throughout the thermodynamical aspects of this analysis, we account for the vertical variations in density. In the dynamical calculations we ignore such variations in ρ. The only important falsification which occurs involves the vertical velocity. If (2.7) were replaced by

$$W_1 = \frac{2R\dot{R}}{(r_0^2 - R^2(t))\rho_{\text{amb}}} \int_0^z \rho_{\text{amb}}(z')\,dz',$$

the result would be rather accurate but we will not bother with such corrections in what follows because we care only about w and related quantities at the boundary-layer level.

In the inner region II and V of figure 6, we take the velocity field to be

$$ru = \phi = r^2\dot{R}/R, \tag{2.9}$$

$$rv = \psi = \Omega(R_0^2 - R^2)\, r^2/R^2. \tag{2.10}$$

The flow into the bottom of this region from the outer boundary layer, III, is given by (2.8) and, above the adjustment region,

$$w_z = -2\dot{R}/R. \tag{2.11}$$

The pressure drop $p(r_0, 0, t) - p(0, 0, t)$ can be obtained from (2.1). In that calculation, the uu_r term contributes nothing, the wu_z term is negligible and, anticipating that $\alpha \leqslant O(\Omega)$, the u_t term is also negligible.

Thus,

$$p(r_0, 0, t) - p(0, 0, t) = \frac{\rho\Omega^2 R_0^2(R_0^2 - R^2)}{R^2}\left(1 + \left[2\ln\frac{r_0}{R} - 1\right]\frac{R^2}{R_0^2}\right), \tag{2.12}$$

where we have neglected terms of order R^2/r_0^2 compared to those of order unity.

According to (2.3), the pressure at $r = 0$, $z = 0$ must depend on the location of C–C and on the fact that the thermodynamic states of the gas both below and above C–C are different from those of the ambient gas near $r = r_0$, which achieved their condition in a more leisurely fashion.

Figure 7 depicts the character of the state changes in the central column of gas. At the time t to which the picture applies, gas which in the ambient state was at ϵ is now at ϵ'; all ambient gas characterized by states to the left of ϵ has flowed up and out from $r = 0$. Gas originally at δ is now δ' and gas now at sea level has the state depicted by O. Each curve $\epsilon\epsilon'$, $\delta\delta'$, and $\alpha\alpha'$, is a moist adiabat corresponding to an expansion from ϵ, δ and α, respectively. Thus, at time t, the column of gas has a sequence of states lying on the curve $O\alpha'\delta'\epsilon'$. The location of O is determined by the fact that the altitude at ϵ' is H, the altitude at O is zero and by (2.3).

It would be a horrendous task to calculate accurately the precise value of $p(0, 0, t)$ for each time t and it would be impossible to do so without some assumption (or calculation) of the vertical velocity dependence on r and t throughout $r < R$. However, we do know that when C–C (figure 1) reaches the top of the domain, the central gas column has states which lie on $L\alpha'b$ and the zero altitude pressure at L is easy to calculate. Accordingly (and tentatively), we merely approximate this hydrostatic determination of the centre-line sea-level pressure by pretending that

$$\frac{p_\alpha - p_0(t)}{p_\alpha - p_L} = \frac{p_\alpha - p_{\alpha'}}{p_\alpha - p_b}. \tag{2.13}$$

Since $(p_\alpha - p_{\alpha'})/(p_\alpha - p_b)$ is approximately the fraction of the weight of the column which is made of new gas [the exact fraction is $(p_0 - p_{\alpha'})/(p_0 - p_b)$], the foregoing approximation merely asserts a proportionality between the pressure drop at the bottom and the fractional weight of 'new' gas in the column. The proportion is so chosen that it is correct when no new gas has entered and is correct when the column is filled by new gas. For a typical atmosphere (Jordan 1957), $p_\alpha - p_L = 50$ millibars and $p_\alpha - p_b \simeq 850$ millibars. Thus,

$$\pi R^2(p_\alpha - p_0) = \frac{\pi R^2}{17}(p_\alpha - p_{\alpha'}) = \frac{\rho_0 g}{17}\int_0^t Q\,dt,$$

i.e.

$$\frac{d}{dt}[\pi R^2(p_\alpha - p_0)] = \rho g Q/17. \tag{2.14}$$

We notice that the term $(p_\alpha - p_0)$ of (2.14) and figure 6 is the same quantity as that denoted by $p(r_0, 0, t) - p(0, 0, t)$ in (2.12) and that Q is defined by (2.8);

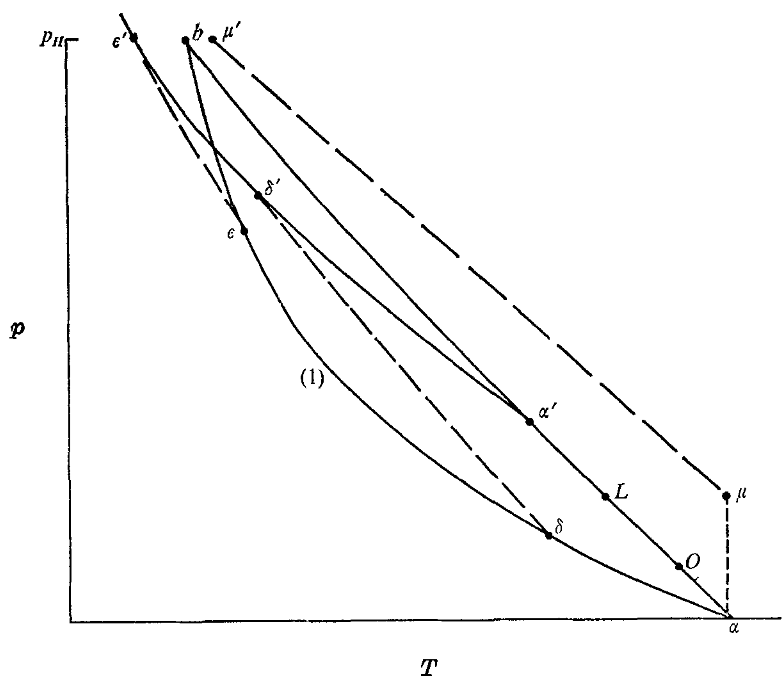

FIGURE 7

thus, we could substitute the information given by (2.12) and (2.8) into (2.14) if it were not for the presence of the factor $M \equiv 1 + [2\ln(r_0/R) - 1]\,R^2/R_0^2$ in (2.12). With that factor the left side of (2.14) would not be proportional to $e^{\alpha t}$ and the right side would.

However, it is no more an arbitrary decision to write

$$\frac{p_\alpha - p_0}{p_\alpha - p_L} = \left[1 + \left(2\ln\frac{r_0}{R} - 1\right)\frac{R^2}{R_0^2}\right]\frac{p_\alpha - p_{\alpha'}}{p_\alpha - p_b} \tag{2.15}$$

than it was to write (2.13). The factor M differs appreciably from unity only when the storm is still weak and there are effects in these early stages which also imply a slower development and which we discuss later. Accordingly, we replace (2.14) by

$$\frac{d}{dt}\left[\frac{\pi R^2(p_\alpha - p_0)}{1 + \left(2\ln\frac{r_0}{R} - 1\right)\frac{R^2}{R_0^2}}\right] = \tfrac{1}{17}\rho g Q, \tag{2.16}$$

and we use (2.12) and (2.8) in (2.16) (recalling that we have already chosen $R_0^2 - R^2 = R_0^2 e^{\alpha t}$) to obtain

$$\frac{\alpha}{\Omega} = \frac{g(\nu\Omega)^{\frac{1}{2}}}{17\Omega^3 R_0^2}\left(1 - \frac{6\alpha}{7\Omega}\right). \tag{2.17}$$

For the numbers of interest ($\Omega = \frac{1}{16}\,\mathrm{h}^{-1}$, $g = 32\,\mathrm{ft/sec^2}$, $\nu = 10^5\,\mathrm{cm^2/sec}$, $R_0 = 200$ miles) this becomes

$$\frac{\alpha}{\Omega} = 7{\cdot}5\left(1 - \frac{6\alpha}{7\Omega}\right)$$

and
$$\alpha \simeq \Omega. \tag{2.18}$$

We might also have decided to ignore the fact that $1 + [2\ln(r_0/R) - 1]R^2/R_0^2$ is a function of time and solve (2.12), (2.8) and (2.14) for α. When that is done, we get

$$\frac{\alpha}{\Omega} = \frac{7{\cdot}5}{1 + \left(2\ln\dfrac{r_0}{R} - 1\right)\dfrac{R^2}{R_0^2}}\left(1 - \frac{6\alpha}{7\Omega}\right),$$

and, even when $R = R_0$,
$$\alpha \simeq 0{\cdot}8\Omega.$$

Thus, the very insensitivity of the result to the size (or changing size) of the coefficient of $(1 - 6\alpha/7\Omega)$ substantiates the hypothesis that the precise form of the proportionality (illustrated by (2.14) and (2.16)) does not matter much. Furthermore, it is comforting to note that substantial changes in the value of ν cannot change this result for α in any significant way.

Accordingly, we conclude that, with the foregoing mechanisms, the storm would intensify with an e-folding time, T, which is about 16 h. At the early stages, the time scale would really be longer; during such early stages, the effects of the vertical transport mechanisms which were discussed in the introduction (i.e. cumulus convection, turbulent diffusion, radiation) will produce an enthalpy distribution in the updraft region which is intermediate between (i) that which is described by figure 6 and whose effect is approximated by (2.15), and (ii) the ambient profile itself. With such central states, $p_\alpha - p_0$ is smaller than the foregoing theory says it is and the early development time is longer.

3. The eye

It must be quite evident that the rigid body rotation in the developing core (which we used *only* to help calculate the radial pressure drop) is probably a poor approximation to reality. In the first place, the boundary layer under such a rigidly rotating flow cannot transport as much fluid as is supplied from $r > R$. Furthermore, the detailed pressure–velocity balance in that region with the peculiar enthalpy distribution carried by the inflow into the region of rising air almost surely implies a different set of details in the inner swirl pattern. It may even be that a disproportionate share of the fluid goes up an annular region even in the early stages. If, for example, this were so (see figure 8), the pressure at A under the annulus would become lower than the central pressure, an anticyclonic swirl would necessarily develop inside the annulus (i.e. in B), high altitude air would sink and, again, the warm dry eye would form.

The point which must be emphasized here is that the details of the initial 'filling' process cannot have profound effects. Whatever happens early, the inertial overshoot of the winding up of the cyclone *must* provide for an insertion of high enthalpy dry air and no alternatives to the first picture can do anything to alter this except to provide for an earlier initiation of that insertion. Thus, the

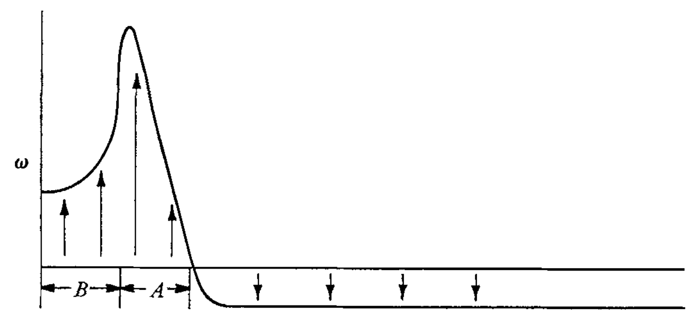

FIGURE 8

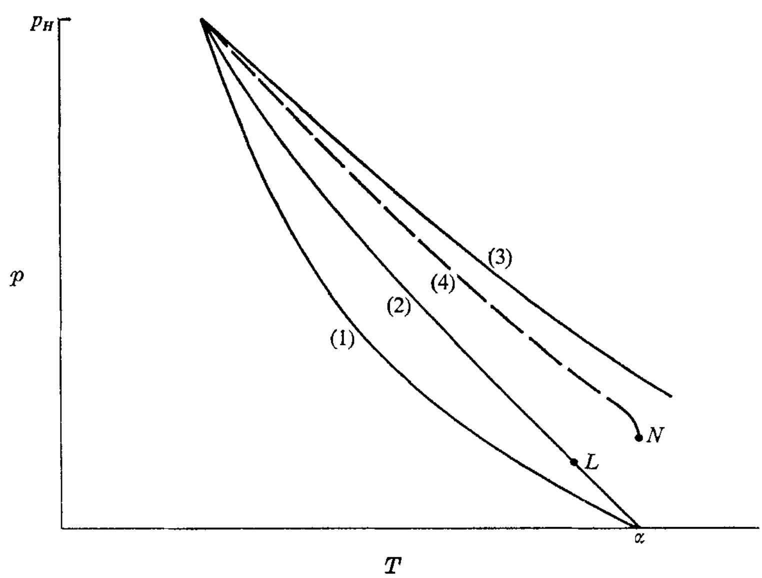

FIGURE 9

thermodynamic states in the eye must lie between the moist adiabat and the dry adiabat obtained by recompressing the air from state b of figure 7. This is indicated, schematically again, in figure 9.

Furthermore, perturbations of the steady-state eye will have the following history. Suppose the central air loses heat so that it is heavier than at equilibrium. An anticyclonic swirl must develop, the boundary layer will pump air radially outward and the air in the column will sink. This lightens the column and the situation is stable. A sign reversal of the initial perturbation does not change this conclusion. Thus, it becomes unnecessary to examine the details in this region. However, we can try to estimate the time scale for the eye formation. When the central column has filled with moist adiabat air and when R differs by a small amount from its equilibrium value, R_e, the excess of $\int_0^{r_0} V^2/r\,dr$ must be compensated by $\partial/\partial t\int_0^{r_0} u_t\,dr$. Thus

$$\int_0^{r_0}\frac{V^2}{r}\,dr_{\text{equil}}-\int_0^{r_0}\frac{V^2}{r}\,dr_{\text{transient}}=\int_0^{r_0}u_{tt}\,dr. \tag{3.1}$$

With $R^2 = R_e^2+\Delta R^2$, with

$$u=\begin{cases}\dfrac{r\dot{R}}{r} & \text{in}\quad r^2<R^2=R_e^2+\Delta R^2,\\[2mm] \dfrac{R\dot{R}}{r}\dfrac{r_0^2-r^2}{r_0^2-R^2} & \text{in}\quad r^2>R^2,\end{cases}$$

with

$$v=\begin{cases}(R_0^2-R^2)\dfrac{r}{R^2} & \text{in}\quad r<R,\\[2mm] \dfrac{(R_0^2-R^2)}{r}\dfrac{r_0^2-r^2}{r_0^2-R^2} & \text{in}\quad r>R,\end{cases}$$

and with $\Delta R^2 \ll R_e^2$, (3.1) becomes

$$\ln\frac{r_0}{R_e}(\Delta R^2)_{tt}=-\Omega^2\frac{R_0^4}{R_e^4}(\Delta R^2). \tag{3.2}$$

The time scale, T, so obtained is *very* short, i.e. $T \simeq 4R_e^2/\Omega R_0^2 = O(20\,\text{min})$.

Probably the formation starts sooner than indicated in the overshoot discussion; that would render this time, T, meaningless. However, it is clear that inertial effects do not inhibit an enormously rapid transition from a storm without a well-formed eye to one with such an eye.

4. State changes in $r \gg R$

As the storm develops, $w(r,z,t)$ becomes more and more negative in the outer regions of the storm. More precisely, that part of the flow into the boundary layer which provides the central flushing is given by

$$-w(r,\infty,t)=\frac{R_0^2-R^2}{r_0^2}(\nu\Omega)^{\frac{1}{2}}\left(1-\frac{6\alpha}{7\Omega}\right). \tag{4.1}$$

This downdraft enters into the balance which controls the enthalpy distribution but it implies no significant modification of the foregoing results. Even in the final quasi-steady state, the downdraft is very weak, i.e.

$$-w_{\text{steady}}(r,\infty)=\frac{R_0^2-R_e^2}{r_0^2}(\nu\Omega)^{\frac{1}{2}}, \tag{4.2}$$

but is still much larger than that of (4.1).

But even at this phase of the storm, there is a negligible change from the ambient distribution (CHG). At the very much lower downdrafts which accompany the early development, the perturbations of the ambient state at large r are even smaller and have no influence on the sequence of events.

There is one feature of both the developing storm and the mature storm which needs to be spelled out more carefully even though its consequences are negligible. Throughout the discussion of the thermodynamics it has been taken for granted that stagnation enthalpy was conserved in the boundary layer and that the state of the updraft air coming from the bottom of the boundary layer should be on the moist adiabat which starts from *ambient* sea-level pressure and temperature. In fact, of course, the air at the foot of the updraft streamline nearest the centre must have sea-level temperature and the *local* sea-level pressure.† That is, if no mixing occurred during its ascent, the states of this air would lie along the moist adiabat $\mu\mu'$ of figure 7. Actually, of course, very little fluid is fed into the updraft by this most central stream tube and the air which does travel via this route must mix with the more plentiful faster moving air. Accordingly, one could modify all of the foregoing as well as the analysis of (CHG) to get even higher limiting velocities, but it is very doubtful that such a modification is worth the effort. The enthalpy discrepancy is small and mixing would certainly negate this effect.

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† This fact does not imply an enhanced heat and moisture transfer rate. The moisture mass per unit volume at sea level still depends only on sea surface temperature but, since $\rho < \rho_{\mathrm{amb}}$, the moisture per unit mass of air is augmented as shown (schematically) in figure 7.